

LECTURE 5

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Eg. $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 2 & 4 \\ 1 & 3 & -3 \end{bmatrix}$

Inverse of matrix

$$A = IA$$

$$\begin{bmatrix} 1 & 2 & -1 \\ 2 & 2 & 4 \\ 1 & 3 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$\begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1/2 & 0 \\ -2 & 1/2 & 1 \end{bmatrix} A$$

$$R_1 \rightarrow R_1 - 5R_3$$

$$R_2 \rightarrow R_2 + 3R_3$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 9 & -3/2 & -5 \\ -5 & 1 & 3 \\ -2 & 1/2 & 1 \end{bmatrix} A$$

$I = BA$ $B \rightarrow A^{-1}$

$$1) \begin{bmatrix} 2 & 3 & -4 \\ 0 & -4 & 2 \\ 1 & -1 & 5 \end{bmatrix}$$

Ans

$$\begin{bmatrix} \frac{10}{46} & \frac{11}{46} & \frac{5}{23} \\ -\frac{1}{23} & \frac{7}{23} & \frac{2}{23} \\ -\frac{2}{23} & \frac{5}{46} & \frac{1}{23} \end{bmatrix}$$

Inverse: $A^{-1} = \frac{\text{Adj } A}{|A|}$ $|A| = \det(A)$

$\text{Adj } A = [C_{ij}]^T$ $C_{ij} = (-1)^{i+j} A_{ij}$

$A_{ij} \rightarrow$ submatrix of A obtained after deleting i th row & j th column.

$\det A = a_{11} A_{11} - a_{12} A_{12} + \dots + (-1)^{n+1} a_{1n} A_{1n}$

Eg: $\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix} = 1 \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} - 1 \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} + 2 \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix}$
 $= 1(2-1) - 1(1-2) + 2(1-4)$
 $= 1 + 1 - 6 = -4$

Eg: $\begin{bmatrix} \quad \quad \quad \end{bmatrix}_{4 \times 4}$

Cramer's Rule: $2x + y + z = 1$, $x + 2y + z = 1$, $x + y + 2z = 1$

$A \rightarrow n \times n$ square matrix $x = [x_1 \dots x_n]$

$b = [b_1 \dots b_n]$

$x_i = \frac{\det [A_i(b)]}{\det A}$

$A_i(b) \rightarrow$ is obtained by replacing i th column of A by column matrix b

$b = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$

$A_1(b) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$

$A_2(b) = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$$A_1(b) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\det [A_1(b)] = 1$$

$$A_2(b) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\det [A_2(b)] = 1$$

$$A_3(b) = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\det [A_3(b)] = 1$$

$$x = \frac{\det [A_1(b)]}{\det A} = \frac{1}{4}$$

$$y = \frac{\det [A_2(b)]}{\det A} = \frac{1}{4}$$

$$z = \frac{\det [A_3(b)]}{\det A} = \frac{1}{4}$$

Eg $\Rightarrow$ 4×4

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$$

$$\det A = 1 \begin{vmatrix} 6 & 7 & 8 \\ 10 & 11 & 12 \\ 14 & 15 & 16 \end{vmatrix} - 2 \begin{vmatrix} 5 & 7 & 8 \\ 9 & 11 & 12 \\ 13 & 15 & 16 \end{vmatrix} + 3 \begin{vmatrix} 5 & 6 & 8 \\ 9 & 10 & 12 \\ 13 & 14 & 16 \end{vmatrix} - 4 \begin{vmatrix} 5 & 6 & 7 \\ 9 & 10 & 11 \\ 13 & 14 & 15 \end{vmatrix}$$

Note: 3×4 , unique sol. (only) by Cramer's rule

Properties of Inverse:

- 1) A & B are invertible then $(AB)^{-1} = B^{-1}A^{-1}$
- 2) $(A^T)^{-1} = A^{-1}$

$$2) (A^T)^{-1} = A$$

$$3) (cA)^{-1} = \frac{1}{c} A^{-1}$$

$$4) (A^K)^{-1} = (A^{-1})^K$$

$$5) (A^T)^{-1} = (A^{-1})^T$$

Properties of determinant:

$$1) \det(AB) = \det A \cdot \det B$$

$$2) \det(KA) = K^n \det A \quad \text{A - square of order } n$$

$$3) \text{ If } A \text{ has a row / column as zero, } \det A = 0$$

$$4) \text{ If } A \text{ has two row / column same, } \det A = 0$$

$$5) \det(A^T) = \det A$$

$$6) \text{ B is matrix obtained by Interchanging one row of A, then } \det B = -\det A$$